Sponsored Search Auctions: Recent Advances and Future Directions

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Sponsored search has been proven to be a successful business model, and sponsored search auctions have become a hot research direction. There have been many exciting advances in this field, especially in recent years; while at the same time, there are also many open problems waiting for us to resolve. In this paper, we provide a comprehensive review of sponsored search auctions, in hope to help both industry practitioners and academic researchers to get familiar with this field, to know the state of the art, and to identify future research topics. Specifically, we organize the paper into two parts. In the first part, we review research works on sponsored search auctions with basic settings, where fully rational advertisers without budget constraints, pre-known click-through rates (CTRs) without inter-dependence, and exact match between queries and keywords are assumed. Under these assumptions, we first introduce the generalized second price (GSP) auction, which is the most popularly used auction mechanism in the industry. Then we give the definitions of several well studied equilibria, and review the latest results on GSP's efficiency and revenue in these equilibria. In the second part, we introduce some advanced topics on sponsored search auctions. In these advanced topics, one or more assumptions made in the basic settings are relaxed. For example, the CTR of an ad could be unknown and dependent on other ads; keywords could be broadly matched to queries before auctions are executed; advertisers are not necessarily fully rational, could have budget constraints, and may prefer rich bidding languages. Given that the research on these advanced topics is still immature, in each section of the second part, we provide our opinions on how to make further advances, in addition to describing what have been done by researchers in the corresponding direction.

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1. INTRODUCTION

Online advertising has become a key business model for Internet companies. According to the report of IAB, the full-year online advertising revenue reaches \$36.57 billion dollars in 2012, which corresponds to a 15% increase from the \$31.74 billion dollars reported in 2011. As a major format of online advertising, sponsored search (a.k.a. search advertising) accounts for 46.3% of the total revenue in 2012 (i.e., \$16.9 billion). It is very clear that sponsored search has already been, and will continue to be very important to many companies, because of the great success of commercial search en-

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gines (e.g., google.com, bing.com, and baidu.com) as well as the huge number of search users, if they want to make effective online marketing.

In addition to the great industrial success, sponsored search has also attracted a lot of attention from the research community and many research papers have been published on the related topics. These papers have their diverse focuses, touching different aspects of sponsored search.

A sponsored search system usually contains two main components: an offline component that provides an interface to advertisers for them to create ad campaigns, select keywords, submit bids, review campaign performance, and make adjustment, and an online component that receives queries from a user, selects relevant ads from the ad database, ranks the selected ads, displays top-ranked ads to the user, and charges the advertisers when their ads are viewed or clicked by the user. An auction mechanism is usually used in the online component to determine the ranking of the ads and the payment of the advertisers. Different pricing models can be adopted in sponsored search auctions, such as pay-per-mille or pay-per-impression (PPM), pay-per-click (PPC), and pay-per-action (PPA). In PPM auctions, advertisers are charged for ad impressions; in PPC auctions, advertisers are charged for clicks on their ads; in PPA auctions, advertisers are charged only if there are user actions/conversions led to by the ad. Today, PPC auctions are dominant in the industry of sponsored search and are also the major focus of our paper.

The research works in the literature have been concerned with different aspects of the sponsored search systems, including keyword suggestion [Joshi and Motwani 2006; Abhishek and Hosanagar 2007; Fuxman et al. 2008; Wu et al. 2009; Ravi et al. 2010], query expansion and ad selection [Radlinski et al. 2008; Broder et al. 2008; Broder et al. 2009; Wang et al. 2009; Broder et al. 2009; Choi et al. 2010], click prediction [sha ; Sculley et al. 2009; Hillard et al. 2010; Graepel et al. 2010; Cheng and Cantú-Paz 2010; Zhu et al. 2010; Kim et al. 2011; Xiong et al. 2012], campaign optimization [Borgs et al. 2007; Feldman et al. 2007; Abrams et al. 2007; Muthukrishnan et al. 2007; Zhang et al. 2012; Ding et al. 2013a], and auction mechanism design [Lahaie 2006; Aggarwal et al. 2006; Varian 2007; Lahaie et al. 2007; Edelman and Ostrovsky 2007; Edelman et al. 2007; Aggarwal et al. 2008; Devanur and Kakade 2009; Goel and Munagala 2009; Aggarwal et al. 2009; Ghosh and Sayedi 2010; Edelman and Schwarz 2010; Milgrom 2010; Gatti et al. 2012]. Fain and Pedersen [2006] provided a brief review of sponsored search, and Broder et al. created a course[1] at Stanford university, called Introduction to Computational advertising, on this topic.

The focus of our paper is relatively narrow, which is on the auction mechanisms and their game-theoretic analysis in the context of sponsored search. The motivation is that in our opinion, it is the auction that makes sponsored search different from its sister fields, such as Web search; it is also the auction that connects sponsored search to economics and makes it an interesting inter-discipline.

The history of sponsored search auctions can be dated back to 1997, when Overture first adopted a generalized first price auction mechanism (GFP) in its sponsored search system [Edelman and Ostrovsky 2007; Edelman et al. 2007; Lahaie et al. 2007; Jansen and Mullen 2008]. In this auction mechanism, each advertiser bids on several keywords. When a user searches for a query, the ads that match the query keyword are ranked in the descending order of their bid prices, and the top k ads (suppose there are k ad slots in total) are shown to the user. If an ad is clicked by the user, the corresponding advertiser needs to pay his/her bid price.

Later on, it was found that the GFP mechanism is highly unstable [Edelman et al. 2007; Jansen and Mullen 2008], and is therefore not desirable for both the advertisers

[1] http://www.stanford.edu/class/msande239/

and the search engine. To tackle this problem, Google introduced the generalized second price auctions (GSP), and achieved great practical success. Similar to GFP, GSP ranks ads in the descending order of their bid prices. The difference lies in that an advertiser does not pay his/her own bid price, but instead the bid price of the advertiser whose ad is ranked immediately lower than his/her ad. In some implementations of GSP, the click-through rates (CTR) of the ads are also considered in both ranking and pricing of the ads. Over the years, GSP has become the golden standard in the industry of sponsored search, and helps the search engine companies to generate revenues of tens of billions of dollars.

Inspired by the practical success of GSP (and that of the sponsored search industry), many researchers from different disciplines, such as Economics, Business, Management Science, and Computer Sciences, have started to investigate the theoretical properties of GSP and to refine GSP or design new auction mechanisms in order to address the problems with GSP (e.g., the ideal assumptions on CTRs and advertisers' behaviors, without considering broad match and budget constraints). As a result, there have been quite a lot of papers on the mechanism design and game-theoretic analysis for sponsored search auctions, especially in recent years. This motivates us to write a survey paper on the topic. The goal of our paper is to provide a simple yet comprehensive description of the sponsored search auctions, and make discussions on the challenges and opportunities (including future research directions) regarding this important field.

According to our survey, the studies on sponsored search auctions can be categorized into two classes based on their settings and assumptions.

The works in the first class (which will be reviewed in Part I of our paper) make some basic assumptions when designing and analyzing an auction mechanism. For example, the advertisers are assumed to have full rationality, have no budget constraints, and their ads are assumed to have known and independent CTRs. Under these assumptions, the following research questions are raised and answered:

(1) Can an auction mechanism achieve certain stable outcomes? What kinds of properties do such stable outcomes possess? These questions are regarding the equilibria of the auction and will be discussed in Section 3.
(2) Can an auction mechanism maximize the social welfare/efficiency? If not, can it achieve a constant fraction of the optimal efficiency? These questions are regarding the allocation rule of the auction, and will be discussed in Section 4.
(3) Can an auction mechanism maximize the revenue of the auctioneer? How to design a revenue-maximizing auction mechanism? These questions are regarding both the allocation and pricing rules of the auction, and will be discussed in Section 5.

Due to the assumptions adopted by the works in the first class, the analysis on the existence of equilibrium, social efficiency, and revenue can be conducted in a relatively easy and neat way. However, these assumptions usually do not hold in real-world sponsored search systems, and therefore we are faced with a gap between theory and practice. The works in the second class (which will be reviewed in Part II of our paper) try to bridge this gap by adopting relaxed assumptions.

(1) Ideal assumptions on known and independent CTRs are clearly violated in practice. No search engines can know the exact CTRs of the ads, and the CTRs of the ads in the same search session can actually affect each other. Sections 6 and 7 are devoted to estimating the CTRs and modeling the dependence of the CTRs respectively.
(2) In practice, most advertisers have their budget constraints, and the search engines should not continue to show ads from an advertiser and to charge him/her when

he/she runs out of budget. In this situation, a good auction mechanism should view sponsored search as a budgeted allocation problem. In Section 8, we will review some existing works on this topic.

(3) Real sponsored search auctions are triggered by search queries, but advertisers' bids are placed on keywords (instead of directly on queries). Search engines usually need to match a query to multiple bid keywords, which is referred to as *broad match*. Broad match involves ads that bid on different keywords in the same auction, which destroys the direct relationship between value and bid in the standard auction theory. In this situation, the previously obtained theoretic properties of an auction mechanism may change dramatically. We will discuss this issue and review related works in Section 9.

(4) In practical sponsored search systems, there are billions of queries and keywords, millions of advertisers and ads. Under such a complex situation, it is almost impossible for any advertiser to collect full information and take the best response to maximize his/her payoff. It is a challenging problem how to model advertisers' behaviors and perform reasonable and insightful analysis without the assumption of full rationality. We will discuss this issue in Section 10.

(5) As the sponsored search industry evolves over time and becomes more and more mature, advertisers have obtained more knowledge about sponsored search auctions and may want a more expressive bidding language to describe their preferences. Section 11 focuses on rich bidding languages for sponsored search auctions, and their impact on the performances of existing auction mechanisms.

Overall, this paper focuses on the game-theoretic aspects of sponsored search auctions. We review the latest results in this field, discuss several advanced topics, and provide our opinions about future research directions. We sincerely hope that by reading this paper, the readers can get a relatively comprehensive picture about sponsored search auctions, and can be inspired to perform some research in this important area. However, we understand that this is not an easy task; we would have been very happy if we can partially achieve the goal through our paper, and we would like to suggest the readers to refer to [Fain and Pedersen 2006; Lahaie et al. 2007; Jansen and Mullen 2008; Muthukrishnan 2008; Yao and Mela 2009] for a more comprehensive understanding of the entire area of sponsored search. In particular, Maillé et al. [2012] provide an overview on sponsored search auctions from another angle and categorize existing work into three classes according to the participating entities, i.e., the search engine, the advertisers, and the users of the search engine.

Part I: Sponsored Search Auctions with Basic Settings

In the first part (Sections 2-5) of the paper, we will introduce the theoretical studies on sponsored search auctions in the following basic settings:

— *Full rationality*: the bidders are assumed to have well-defined utilities, and have the necessary information and computational power to maximize their utilities.
— *Unlimited budget*: the auctioneer is assumed to be able to adopt the optimal auction outcome (allocation and payment) without being constrained by the budgets of individual bidders.
— *Exact match*: all the bidders under investigation are assumed to bid on the same keyword (which is exactly the query), and one does not need to consider the mix of ads from different bid keywords in the auction.

— *Known and independent CTRs*: the CTR of each ad is assumed to be known in advance and independent of other ads; furthermore, it is usually assumed that the CTR is separable and can be decomposed into an ad-specific term and a position-discount term.
— *Single-dimensional Bid*: a bidder is only allowed to submit a single-dimensional bid to express his/her preferences.

2. GENERALIZED SECOND PRICE AUCTIONS

While a number of auction mechanisms can be applied to sponsored search, it is of no doubt that the Generalized Second Price auction (GSP) is the one that attracts most research attention. This is simply because GSP is the most popularly used auction mechanism by commercial search engines today.

To better illustrate the GSP mechanism, let us introduce some notations first.

(1) Assume that there is a set of n bidders, who compete for a set of k ad slots. Here slot 1 corresponds to the slot on the top of the list and slot k is the slot on the bottom. Typically, we have $n > k$ since it is usually the case that many bidders are competing for a limited number of ad slots.
(2) Each bidder i has a private value v_i, expressing the maximum price he/she is willing to pay if his/her ad is clicked.
(3) In order to participate in the auction, bidder i is required to submit a bid b_i. b_i is a proxy of the value v_i but might not exactly equal v_i due to the strategic behaviors of the bidders. Submitting a bid b_i guarantees that bidder i will not be charged by a price higher than b_i. We usually put the bids of all the bidders into a vector for ease of representation, and call it a bid profile or strategy profile: $b = (b_1, \cdots, b_n)$.
(4) We use $CTR_{i,s}$ to represent the CTR of the ad of bidder i when it is placed at position s. As mentioned in the beginning of this part, we impose the assumption that the CTR is known in advance and is separable in most of our discussions. That is, the CTR can be decomposed into a position-discount term θ_s (we call θ_s slot-CTR for ease of reference, which denotes the probability that a user will notice the ad shown at slot s) and an advertiser-specific term q_i (which we call ad-CTR, denoting the probability that a user will click on an ad after he/she notices it), i.e., $CTR_{i,s} = q_i\theta_s$.

GSP consists of a ranking rule and a pricing rule. The ranking rule determines the allocation of the ad slots to the bidders; the pricing rule determines the payment of each bidder when his/her ad is clicked. In the literature, GSP has different implementations of its ranking rule (as shown below), while the pricing rule is almost the same: each bidder pays the minimum bid that he/she could have offered in order to be assigned the current rank position. The ranking rule is usually specified by a ranking function $y(q_i, b_i)$, which takes the ad-CTR and bid as inputs and outputs a score for ranking: the ad with the largest score wins the first slot, the ad with the second largest score wins the second slot, so on and so forth. Two popular choices of the ranking function are as below.

(1) $y(q_i, b_i) = b_i$, which ranks the ads in the descending order of their bids. Assume that bidders are numbered in the descending order of their bids. Then bidder $i \leq k$ wins slot i and will be charged a price of b_{i+1} for each click. GSP with this ranking function is referred to as the rank-by-bid GSP.
(2) $y(q_i, b_i) = q_ib_i$, which ranks the ads according to the products of the ad-CTRs and bids. Assume that bidders are numbered in the descending order of q_ib_i. Then bid-

Table I. Table of Notations

Symbol	Explanation
n	number of bidders
k	number of ad slots
v_i	per-click value of bidder i
v	value profile, i.e., $(v_1, ..., v_n)$
b_i	bid of bidder i
p_i	per-click payment of bidder i
$CTR_{i,j}$	CTR of the ad of bidder i when it is placed at slot j
q_i	ad-CTR of the ad of bidder i
θ_i	slot-CTR of slot i
b_{-i}	bids of all other bidder except i
s^*	utility-maximization slot for bidder i given b_{-i}
$OPT(v)$	optimal social welfare given value profile v
$SW(b, v)$	social welfare associate with b, given v
$b_i(v_i)$	bidding strategy of bidder i based on his/her value v_i
γ_k	θ_k/θ_{k-1}
F	joint distribution of all the bidders' values

der $i \leq k$ wins slot i, and his/her per-click payment p_i is

$$p_i = \frac{q_{i+1} b_{i+1}}{q_i}.$$

GSP with this ranking function is referred to as the rank-by-revenue GSP.

The ranking function of GSP can also take other forms, which may depend on the CTR [Lahaie and Pennock 2007; Thompson and Leyton-Brown 2013; Roberts et al. 2013], and may also be totally independent of the CTR [Aggarwal et al. 2006]. We will mention several other forms of the ranking function in Section 5.

It is well known that the GSP mechanism is not truthful, i.e., bidders will have the incentive to strategically shade their bids in order to gain certain advantages. The Vickrey-Clarke-Groves (VCG) mechanism [Groves 1973], which is well known for its truthfulness and social welfare maximizing properties, is usually used as a reference to make comparisons when conducting theoretical analysis on GSP. When adapted to sponsored search auctions, the allocation results of VCG are the same as the rank-by-revenue GSP mechanism if the CTRs are separable[2], but the payment is different. Assume that bidders are numbered in the decreasing order of $q_i b_i$. In VCG, bidder $i \leq k$ wins slot i, and his/her per-click payment is

$$p_i = \sum_{j=i}^{k} \frac{\theta_j - \theta_{j+1}}{\theta_i} \frac{q_{j+1}}{q_i} b_{j+1},$$

where we assume that $\theta_{k+1} = 0$.

For ease of descriptions, we summarize the frequently used notations in Table I. These notations will not only be used in this section, but also the other sections of our paper. Furthermore, it is noted that "advertiser", "bidder", and "player" are almost interchangeable concepts in our paper, although we choose to use one or another of them more frequently in some specific contexts.

3. EQUILIBRIUM CONCEPTS

The bids are proxies of the values. From a game-theoretic perspective, an ideal situation for the auctioneer is to ensure that the bidders have no incentive to misreport

[2]If the CTRs are not separable, a new auction mechanism called *Laddered Auction* is designed to ensure that truthful bidding is a dominant strategy [Aggarwal et al. 2006].

their values since this would eliminate the possibility of potential manipulations of the auction mechanism by the bidders. However, as aforementioned, GSP cannot guarantee truthful bidding, i.e., bidders can be better off by shading their values [Edelman et al. 2007]. In this situation, it makes more sense to study the space of equilibrium bid profiles. In the literature, a number of equilibrium concepts have been used in the theoretical study of GSP. We will make brief introductions to them in this section. For sake of simplicity and without loss of generality, we assume that the ad-CTR q_i is the same for all the ads and ignore it. The equilibrium concepts defined in this section can be easily extended to the case where ad-CTRs are different.

3.1. Equilibria

We consider equilibria under both full-information and partial-information settings.

3.1.1. Pure Nash Equilibrium under Full-Information Setting. In the full-information setting, the value profile v and slot-CTR θ are fixed and are common knowledge to both the auctioneer and the bidders.

In game theory, a Nash equilibrium [Nash 1951] is defined as a stable outcome of a game, i.e., a situation where no player can improve his/her payoff (utility) by a unilateral strategy change. In the context of sponsored search auctions, we say that a bid profile b is a pure Nash equilibrium if for every bidder i we have

$$u_i(b_i, b_{-i}) \geq u_i(b'_i, b_{-i}), \forall b'_i,$$

where $u_i(\cdot)$ is the utility function of bidder i, b_{-i} is the bids of all the other bidders, and b'_i is an alternative bid of bidder i.

Note that in GSP, a bidder must change his/her slot (through changing the bid) in order to change his/her utility. Therefore, at a Nash equilibrium, no bidder would have an incentive to obtain a different slot. This can be characterized by the following definition.

Definition 3.1. A bid profile is a *pure Nash equilibrium* if for every slot s and for the bidder at this slot, the following inequalities hold:

$$\theta_s(v_s - p_s) \geq \theta_j(v_s - p_j) \qquad \forall j > s,$$

$$\theta_s(v_s - p_s) \geq \theta_j(v_s - p_{j-1}) \qquad \forall j < s,$$

where p_s is the payment of the bidder occupying slot s.

It has been proven that there always exists a pure Nash equilibrium for the GSP mechanism [Edelman et al. 2007; Varian 2007].

3.1.2. Pure Symmetric Nash Equilibrium under Full-Information Setting. A subset of Nash equilibria has been studied in the literature of sponsored search, which is called symmetric Nash equilibria [Varian 2007]. Its definition is given as follows.

Definition 3.2. A bid profile b is a pure *symmetric Nash equilibrium* if it satisfies:

$$\theta_s(v_s - p_s) \geq \theta_j(v_s - p_j) \qquad \forall j, s. \tag{1}$$

According to the above definition, symmetric Nash equilibrium captures the notion that there should be no incentives for any pair of bidders to swap their slots. Varian [2007] proved that there exists a symmetric Nash equilibrium for GSP that achieves the maximal revenue among all Nash equilibria.

Locally envy-free equilibria have been studied in [Edelman et al. 2007], which can be verified to be equivalent to symmetric Nash equibliria.

Definition 3.3. A bid profile b is a pure *locally envy-free equilibrium* if any bidder cannot improve his/her utility by exchanging bids with the bidder ranked one position above him/her:

$$\theta_s(v_s - p_s) \geq \theta_{s-1}(v_s - p_{s-1}) \qquad \forall s. \tag{2}$$

The pure equilibria introduced above can be easily extended to mixed-strategy equilibria[3] by taking expectation over the randomized declaration of bids.

3.1.3. Bayesian Nash Equilibria under Partial-Information Setting. In the partial-information (Bayesian) setting, the value profile is assumed to be drawn from a publicly known (possibly correlated) joint distribution $\mathbf{F}$. A strategy for bidder i is a (possibly randomized) mapping $b_i : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ from his/her value v_i to a bid $b_i(v_i)$. For ease of reference, we use $b(v) = (b_1(v_1), \cdots, b_n(v_n))$ to denote the profile of bids when the corresponding value profile is v.

Definition 3.4. A strategy profile b is a Bayesian Nash equilibrium for a joint distribution $\mathbf{F}$ if, for all i, all v_i, and all alternative strategies b'_i,

$$E_{v_{-i},b}[u_i(b_i(v_i), b_{-i}(v_{-i}))|v_i] \geq E_{v_{-i},b}[u_i(b'_i(v_i), b_{-i}(v_{-i}))|v_i].$$

The above definition says that each bidder maximizes his/her expected utility by bidding in accordance with strategy $b_i(\cdot)$, assuming that the other bidders bid in accordance with strategies $b_{-i}(\cdot)$, where the expectation is taken over the distribution of the other bidder's types conditioned on v_i and any randomness in their strategies.

A necessary and sufficient condition has been given in [Gomes and Sweeney 2012] to guarantee the existence of an efficient Bayesian Nash equilibrium for GSP.

3.2. Convergence to Equilibria

In the previous subsection, we have introduced some equilibrium concepts associated with GSP, however, it is unclear how these equilibria can be achieved in practice. Sponsored search auctions are actually repeated games and advertisers have the opportunity to update their bids according to some strategies. The question is whether reasonable bidding strategies of the advertisers can eventually converge to certain kind of equilibrium of our interests. In this subsection, we make discussions on this issue.

The most-studied bidding strategy is the so-called best-response bidding strategy, which is defined as follows [Cary et al. 2007].

A best-response bidding strategy for bidder j is to choose a bid for the next round of a repeated auction so as to maximize his/her utility u_j, assuming the bids of all the other bidders b_{-j} in the next round remain unchanged as compared to the previous round.

As we know, in GSP, the bid of an advertiser only affects the slot that he/she may be assigned to (and therefore the value he/she can eventually realize), but does not affect his/her payment (due to the second-price rule) as long as his/her slot remains the same. Therefore, the advertiser will obtain the same utility when his/her bid falls into a certain range that corresponds to the same ad slot (which we call the best-response range for ease of reference). In this case, we need to specify which bid in the best-response range is eventually used by the advertiser.

Cary et al. [2007] proposed a *balanced bidding strategy* (or identically *forward looking strategy* in [Bu et al. 2007])as a specific implementation of the best-response

[3]A pure strategy for bidder i is a function $b_i : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ that maps a private value to a declared bid; a mixed strategy maps a private value to a distribution over bids, corresponding to a randomized declaration.

strategy, which is defined as below.

For bidder j, given b_{-j}, with a balanced bidding strategy, he/she will set his/her bid b_j according to $\theta_{s^*}(v_j - p_{s^*}) = \theta_{s^*-1}(v_j - b_j)$, where s^* is his/her utility-maximization slot given b_{-j} and p_{s^*} is the corresponding payment at this slot.

The intuition for the balanced bidding strategy is as follows. On one hand, advertiser j wants to set a larger bid within the best respond range in order to make the advertiser ranked above him/her pay more money. On the other hand, if his/her bid is too large, the advertiser originally ranked above him/her may give up the current position and change the bid to be just a little smaller than advertiser j's bid. This will impose some risk on advertiser j and he/she may want to seek a balance between these two cases.

Cary et al. [2007] showed that the balanced bidding strategy has the following convergence properties. (1) The fixed point of the balanced bidding strategy can be characterized, in which the revenue of the auctioneer equals that in the VCG equilibrium. (2) Assuming that bidders are numbered in the descending order of their values, when there are two slots, the auction system converges to the fixed point after $O(\log((v_2 - v_3)/v_3))$ rounds of bid update; in the multi-slot case, the auction system will converge to the fixed point with the same rate, if only one randomly selected bidder updates his/her bid in each round.

Kominers [2009] generalized the balanced bidding strategy in a dynamic position auction in the presence of web users' search behavior and similar convergence results are obtained. Vorobeychik and Reeves [2007] conducted simulations and showed that the balanced bidding strategy is highly stable, i.e., the additional gain from deviation from it is low when all bidders follow the balanced bidding strategies.

Nisan et al. [2011] gave answer to the question why would we expect the bidders to repeatedly best-responds to others. They proved that in GSP auctions, if all the other bidders are repeatedly best responding, one's best choice is also best response. They term auctions/mechanisms with this property *best-response auctions/mechanisms*.

In addition to the balanced bidding strategy, some other bidding strategies have also been studied in the literatures.

(1) The *altruistic bidding* and *competitor busting* strategies are proposed in [Cary et al. 2007] and [Zhou and Lukose 2007], in which each bidder bids just ϵ lower than the upper bound or larger than the lower bound of the best-response range. However, the auction system may not always converge when these two bidding strategies are adopted. Markakis and Telelis [2010] derived an upper bound for the parameter ϵ in the altruistic bidding and competitor busting strategies in order to ensure that the induced configuration space contains an optimal pure Nash equilibrium.
(2) Liang and Qi [2007] studied three variants of vindictive bidding strategies in conjunction with the cooperative forward-looking bidding strategy, and proved their convergence to some Nash equilibria.
(3) Yao et al. [2012] proposed a weighted joint fictitious play bidding strategy, in which bidders' beliefs are weighted combinations of the empirical distribution of others' bids in the past. This strategy is proven to converge to Nash equilibrium of GSP under certain conditions.

4. EFFICIENCY ANALYSIS

Social efficiency is an important property of an auction mechanism, which measures to what extent the social welfare is optimized by the auction. In this section, we introduce some existing research on the social efficiency of GSP.

Similar to the previous section, here we also assume that the ad-CTRs are the same for all the bidders, and ignore the ad-CTRs (e.g., by setting $q_i = 1, \forall i$) for efficiency analysis. The results can be easily generalized to the case with different ad-CTRs.

4.1. Price of Anarchy

The social efficiency is usually measured[4] by the so-called price of anarchy (PoA), whose definition is given in this subsection.

In the full-information setting, without loss of generality, we assume $v_1 > v_2 > \cdots > v_n$. Then for a given bid profile $b = (b_1, \cdots, b_n)$, the social welfare is $SW(b,v) = \sum_{j=1}^{k} \theta_j v_{(j)}$, where $v_{(j)}$ denotes the value of the bidder ranked at slot j by the allocation rule. On the other hand, the optimal social welfare would be $OPT(v) = \sum_{j=1}^{k} \theta_j v_j$. One usually uses pure PoA (which is defined below) to measure the difference between SW and OPT in the worst pure Nash equilbrium [Koutsoupias and Papadimitriou 1999]:

$$\sup_{v,b \in NE} \frac{OPT(v)}{SW(b,v)},$$

where the supremum is taken over all possible values and corresponding bid profiles that constitute pure Nash equilibria.

Similarly, we can define the PoA for the mixed-strategy Nash equilibria. In this case, a bidding strategy $b_i(v_i)$ is a distribution over declarations.

$$\sup_{v,b(v) \in NE} \frac{OPT(v)}{E_{b(v)}[SW(b(v),v)]},$$

where the supremum is taken over all possible values and corresponding bid profiles that constitute mixed Nash equilibria.

Moreover, we can define the Bayesian Nash PoA in the partial-information setting as follows,

$$\sup_{\mathbf{F},b() \in BNE} \frac{E_v OPT(v)}{E_{v,b(v)}[SW(b(v),v)]},$$

where the supremum is taken over all the distributions of values and corresponding bid profiles that constitute Bayesian Nash equilibria.

4.2. Bounding Price of Anarchy

Pure PoA Bounds. Lahaie [2006] showed that the pure PoA can be upper bounded by $(\min_{i=1,\cdots,k-1} \min\{\gamma_{i+1}, 1-\gamma_{i+2}\})^{-1}$, where $\gamma_i = \theta_i/\theta_{i-1}$ and $\gamma_{k+1} = 0$. This may mean very poor efficiency for arbitrary slot-CTRs. Motivated by the observation that all known examples of poor efficiency occur at the equilibria that involve over-bidding, Leme and Tardos [2010] considered only Nash equilibria among conservative bidders (i.e., bidders that never bid above their values), and obtained an upper bound of 1.618 on the pure PoA. The bound is later improved to 1.282 in [Lucier and Leme 2011; Caragiannis et al. 2011].

[4]Another related metric is price of stability [Roughgarden and Tardos 2007].

Table II. Known PoA Bounds for GSP with Conservative Bidders

Pure Nash equilibria	Mixed Nash equilibria	Bayesian Nash equilibria
1.282	2.310	2.927

Table III. Tight PoA Bounds for GSP with Conservative Bidders

#slots	2	3	4	≥ 5
Tight bound	1.25	1.259	1.259	1.259(?)

Mixed-strategy PoA Bounds. Leme and Tardos [2010] obtained an upper bound of 4 for mixed-strategy PoA for conservative bidders. It is later improved to[5] 2.310 in [Lucier and Leme 2011; Caragiannis et al. 2011].

Bayesian Nash PoA Bounds. Leme and Tardos [2010] obtained an upper bound of 8 for the Bayesian Nash PoA. The bound is later improved to 2.927 in [Caragiannis et al. 2012b].

For clarity and ease of reference, we list the known upper bounds for the PoA of GSP in different settings in Table II.

The lower bounds of PoA are usually achieved by constructing an instance of a GSP auction along with a Nash equilibrium of the appropriate efficiency. When we allow over-bidding, it is not difficult to construct such examples. For conservative bidders, however, lower bounds still remain unclear [Caragiannis et al. 2011]. When the upper bound meets the lower bound, we say the bound is tight. However, it is a challenging issue to determine whether a PoA bound is tight or not. There have been some partial results on the tightness of pure PoA bounds (see Table III). In particular, it is proven that the pure $PoA = 1.25$ when there are two ad slots [Leme and Tardos 2010], $PoA = 1.259$ for three ad slots [Lucier and Leme 2011], and $PoA = 1.259$ for four ad slots [Ding et al. 2013b]. Moreover, it is hypothesized that 1.259 is also the tight pure PoA bound when the number of ad slots is larger than four [Caragiannis et al. 2011; Ding et al. 2013b], and this tight bound can be achieved at the pure Nash equilibrium where the bidder with the largest value is ranked at the bottom slot and the bidder with the i-th ($i > 1$) largest value is ranked at the $(i-1)$-th slot. However, as far as we know, this conjecture has not been formally proven yet.

5. REVENUE ANALYSIS

In this section, we will introduce the theoretical analysis on the search engine revenue for the GSP mechanism. Again, since GSP is not truthful, one needs to investigate the revenue generated at a Nash equilibrium. In the literature, the revenue in equilibria for both the full-information and partial-information settings have been studied.

First, let us have a look at the full-information setting.

Edelman et al. [2007] and Varian [2007] studied the revenue of GSP at the symmetric Nash equilibria (or locally envy-free equilibria) , and showed that the worst-case revenue generated symmetric Nash equilibria is at least as good as the revenue of VCG.

Lucier et al. [2012] studied the revenue of GSP over all Nash equilibria. The finding is that the worst-case revenue in equilibrium of GSP can be arbitrarily bad as compared with VCG. However, if we exclude the payment of the bidder with the largest

[5]The bound of 2.310 also holds for the class of coarse correlated equilibria. There are algorithms where bidders adjust their strategies over time and the average regret for their choices approaches 0 (for more see [Young 2004]).

bid, the revenue for any Nash equilibrium of GSP will be at least half of the revenue of VCG. This result is tight and holds also with any reserve price. Furthermore, Lucier et al. [2012] demonstrated that there exist inefficient, non-envy-free equilibria that can obtain larger revenue than the envy-free equilibria.

Second, we introduce some results for the partial-information setting.

The revenue generated by GSP at Bayesian Nash equilibria has been analyzed when bidders' values are assumed to be drawn from identical distributions [Lucier et al. 2012]. Myerson's Lemma is a useful tool for studying revenue in this setting, which can be rephrased for sponsored search auctions as follows.

At any Bayesian Nash equilibrium of an auction mechanism, we have that, for all $i, E[\alpha_{\sigma(i)} p_i] = E[\alpha_{\sigma(i)} \phi(v_i)]$ *where* $\phi(x) = x - \frac{1-F(x)}{f(x)}$ *is the virtual valuation function,* F *is the distribution over bidders' values,* p_i *is the payment per click of bidder* i*,* $\alpha_{\sigma(i)}$ *is the number of clicks received by bidder* i*, and the expectation is taken with respect to* $v \sim F$*.*

We say that a distribution is *regular* if $\phi(x)$ is a monotone non-decreasing function. For regular distributions, the revenue-optimal mechanism for sponsored search auctions corresponds to running VCG with Myerson's reserve price [Myerson 1981], which is the largest value such that $\phi(r) = 0$. A special class of regular distributions is the *monotone hazard rate* distributions (MHR), for which $f(x)/(1-F(x))$ is non-decreasing.

Lucier et al. [2012] showed that the worst-case revenue for GSP at a Bayesian Nash equilibrium can be arbitrarily bad as compared with VCG: there exists an example where VCG generates a positive revenue but GSP has a Bayesian Nash equilibrium with zero revenue. Furthermore, by assuming that bidders' values are drawn from a specific distribution (i.e., the MHR distribution), Lucier et al. [2012] proved that the GSP auction paired with the Myerson reserve price generates at least a constant fraction (i.e., 1/6) of the optimal revenue in Bayesian Nash equilibria. Caragiannis et al. [2012a] further improved the bound: it is shown that for a regular distribution, there exists a reserve price paired with which GSP can generate at least 1/4.72 of the optimal revenue in Bayesian Nash equilibria; for a MHR distribution, there exists a reserve price paired with which GSP can generate at least 1/3.46 of the optimal revenue in Bayesian Nash equilibria.

Lahaie and Pennock [2007] studied the revenue of GSP with a specific ranking function: $y(q_i, b_i) = q_i^s b_i$, where $0 \leq s \leq 1$ is called a squashing factor. In this situation, the revenue generated at symmetric Nash equilibria is investigated and the following results are obtained: (1) No squashing or other manipulations of ad-CTR can produce the optimal auctions. (2) Simulations reveal that properly tuning the squashing factor can significantly improve revenue. (3) Advertiser satisfaction and user experience could suffer if the squashing factor is too small.

Thompson and Leyton-Brown [2013] studied the revenue of GSP with different implementations of the ranking rule (including squashing factor and different types of reserve prices). The main finding is somehow surprising: unweighted reserve prices are dramatically better than the quality-weighted reserve prices, although the latter have been commonly used by search engines in the past few years. This result is shown to be very robust, appearing in both theoretical analysis and computational experiments.

Roberts et al. [2013] studied the revenue of GSP when the ranking rule takes the following form:

$$y(q_i, b_i) = (g(q_i)b_i - h(q_i))^+,$$

where g and h are arbitrary non-negative functions and independent of b_i. They showed that any ranking rule of this form leads to exactly the same properties as the rank-by-revenue ranking rule: (1) symmetric Nash equilibrium always exists; (2) the ranking in all symmetric Nash equilibria is the same as the ranking when all bidders make truthful bidding; (3) in the symmetric Nash equilibrium with the lowest revenue, each bidder gets the same position and payoff as in the dominant-strategy equilibrium of the game induced by VCG.

In addition to the above results, a very specific ranking function $y(q_i, b_i) = (b_i - r)q_i$, where r is the per-click reserve price, is investigated in [Thompson and Leyton-Brown 2013] and [Roberts et al. 2013]. Thompson and Leyton-Brown [2013] showed that, for single-slot GSP auctions, if all the bidders' values are independently drawn from a uniform distribution, then the allocation function of the optimal auction is identical to the above ranking function. Roberts et al. [2013] showed that, for sufficiently small reserve price r, the lowest revenue for the symmetric Nash equilibrium of the GSP auction subject to the above ranking function is greater than the revenue for any symmetric Nash equilibrium under the standard rank-by-revenue rule with the same reserve price r.

Part II: Advanced Topics

The real-world practices of sponsored search auctions are much more complicated than the theoretical studies introduced in the previous part. In particular, many of the assumptions used in the basic settings are violated in practice. In recent years, people have studied the following advanced topics, which make fewer or weaker assumptions, and therefore should have more practical values.

— *Auctions with unknown CTRs*: In practice the CTRs of ads are unknown to the search engine. The search engines needs to estimate the CTRs through either offline learning or online learning, as studied in [Richardson et al. 2007; Gonen and Pavlov 2007; Wortman et al. 2007; Babaioff et al. 2009; Devanur and Kakade 2009; Xu et al. 2010; Xiong et al. 2012; Gatti et al. 2012; Wang et al. 2013].
— *Auctions with dependent CTRs*: The CTR of an ad is actually dependent on other ads displayed in the same session. The design and analysis of auctions with dependent CTRs have been studied in [Ghosh and Mahdian 2008; Aggarwal et al. 2008; Giotis and Karlin 2008; Kempe and Mahdian 2008; Deng and Yu 2009; Fotakis et al. 2011].
— *Auctions with budget constraints*: Advertisers cannot attend any auction when they run out of budget. Therefore, a good allocation rule should consider the remaining budget of advertisers [Borgs et al. 2005; Abrams et al. 2007; Dobzinski et al. 2008; Azar et al. 2008; Chakrabarty and Goel 2008; Kempe et al. 2009; Hafalir et al. 2009; Ashlagi et al. 2010; Bhattacharya et al. 2010; Van der Laan and Yang 2011; Arnon and Mansour 2011; Koh 2011; Colini-Baldeschi et al. 2012].
— *Broad match auctions*: In practice, search engine matches a query to multiple bid keywords according to a query-keyword bipartite graph. As a result, the ads that bid on different keywords may compete with each other in an auction. The mechanism design and game-theoretic analysis in this setting have been investigated in [Mahdian and Wang 2009; Dhangwatnotai 2011; 2012; Chen et al. 2014].
— *Advertisers' behavior models*: It has been widely observed that advertisers are not fully rational and sometimes do not take the best responses. One needs to build some models to describe advertisers' behaviors and take them into consideration for

auction analysis, as discussed in [Duong and Lahaie 2011; Pin and Key 2011; Xu et al. 2013; He et al. 2013].

— *Auctions with rich bidding languages*: A single dimensional bid might not be sufficient to express the rich demands of advertisers. For example, advertisers may not want to display his/her own ad with those of his/her competitors in the same session; advertisers may have different values for ad impression, click, and conversion; etc. All of this requires the use of richer bidding languages in sponsored search auctions [Aggarwal et al. 2007; Edelman and Lee 2008; Goel and Munagala 2009; Muthukrishnan 2009; Ghosh and Sayedi 2010; Constantin et al. 2011; Hoy et al. 2013].

In the remainder of this part, we will use separate sections to introduce the above advanced topics. As will be seen, there are still quite a lot of open questions regarding these topics, therefore we will provide our own opinions in addition to introducing the existing works.

6. AUCTIONS WITH UNKNOWN CTRS

As mentioned in Part I, many research works on sponsored search auctions have assumed that the CTRs of ads are known in advance. However, this assumption does not hold well in practice: for those ads that have not been shown previously, it is difficult for both advertisers and the search engine to know its CTR; even for the ads that have been shown, one may only get an estimation of their CTRs based on historical click-through data. To solve the problem, we can either build a *click prediction model* to predict the CTR of a given ad, and then substitute the true CTRs in previously designed auctions with the predicted CTRs; or we can adopt online learning to accumulate the knowledge about CTRs, e.g., using multi-armed bandit algorithms, and design some online auction mechanisms.

6.1. Click Prediction Models

Commercial sponsored search systems typically employ a machine learning approach to predict the probability that a user clicks on an ad, and existing works have focused on the two main component of a learning based approach: feature extraction and model selection.

For feature extraction, Dembczynski et al. [2008] and Attenberg et al. [2009] showed that the historical click information of an ad is effective for predicting the future click probability of the ad. However, in practice, there are many ads without adequate historical click-through data (even after aggregation at different levels, e.g., campaign, advertiser, and query levels). To tackle this data sparseness issue, some relevance features have been investigated to improve the accuracy of click prediction, and this type of features is mostly based on the similarity between query and ad, and the quality of the ad [Clarke et al. 2007; Richardson et al. 2007; Zhang et al. 2007; Radlinski et al. 2008; Raghavan and Iyer 2008; Shaparenko et al. 2009].

For model selection, Richardson et al. [2007] proposed a maximum entropy model, which formulates the click probability by combining diverse forms of features; Graepel et al. [2010] introduced the click prediction algorithm used in Microsoft Bing, which is based on a probit regression model that maps discrete or real-valued input features to probabilities. Xu et al. [2010] proposed a Bayesian framework, which is shown to be more effective in incorporating temporal and positional information; Xiong et al. [2012] developed a continuous conditional random fields based model, which succeeds in modeling relational information between different ads. Recently, Wang et al. [2013] analyzed how the psychological needs of users will affect their click behaviors, and built a click prediction model that considers consumer psychology.

6.2. Multi-Armed Bandit Mechanisms

A key challenge for applying online learning in auction mechanism design is the tradeoff between exploration and exploitation. On one hand, one needs exploration to collect more information about CTRs for better allocation and pricing in the future; on the other hand, one needs to avoid losing too much revenue when performing exploration. Multi-armed bandit (MAB) algorithms have been shown effective in dealing with this exploration/exploitation tradeoff in the literature of online learning [Robbins 1952; Berry and Fristedt 1985; Gittins 1989], and have been introduced to sponsored search auctions in recent years. For example, Pandey and Olston [2007] formulated ad placement with budget constraints as a MAB problem and derived a bound for the search engine revenue. However, this work ignores the potential strategic behaviors of the advertisers against the online learning process. Therefore, the resultant auction mechanism may not perform as well as expected in practice.

To tackle the problem, Gonen and Pavlov [2007] discussed how to develop a truthful MAB mechanism for sponsored search. However, as pointed out in [Babaioff et al. 2009; Devanur and Kakade 2009], the mechanism proposed in [Gonen and Pavlov 2007] is actually not as truthful as claimed. Babaioff et al. [2009] and Devanur and Kakade [2009] then characterized the conditions for truthful MAB mechanisms, which is shown as below.

Consider a MAB mechanism $(\mathcal{A}, \mathcal{P})$. Suppose the deterministic allocation rule $\mathcal{A}$ is non-generate, scale-free, and satisfies independence of irrelevant alternatives (IIA). Then $(\mathcal{A}, \mathcal{P})$ is normalized and truthful for some payment rule if and only if $\mathcal{A}$ is pointwise monotone and exploration separated (i.e., independent of the bids).

Furthermore, Babaioff et al. [2009] and Devanur and Kakade [2009] proved that the lower bound on the regret of a truthful MAB mechanism in terms of social welfare and revenue are both $\Omega(T^{2/3})$. As compared with traditional MAB algorithms without truthful restrictions (whose regret is $\Omega(T^{1/2})$), the extra term $\Omega(T^{1/6})$ is called the "price of truthfulness" [Devanur and Kakade 2009]. A very simple MAB mechanism is given in [Babaioff et al. 2009; Devanur and Kakade 2009], which can achieve the lower bound. Specifically, for the first T_0 rounds (exploration), the mechanism explores ads' click information in a robin fashion, and observes the CTR for each ad; in the following rounds (exploitation), the ranking and payment rules are the same as in the rank-by-revenue GSP by regarding the observed CTR as the true CTR.

In addition to the works introduced above, there are also some other research that analyzes the MAB mechanisms either in different notions of truthfulness or in more detailed settings. For example, Babaioff et al. [2010] derived a regret bound for randomized MAB mechanism which is truthful in expectation over random seeds and satisfies additional individual rational conditions. The lower bound for the regret is $\Omega(T^{1/2})$, which matches the lower bound of MAB algorithms without truthful restrictions. Nazerzadeh et al. [2008] proposed a sampling-based online learning algorithm, which is approximately truthful and the regret can approach 0. However, the corresponding regret analysis remains open. Wortman et al. [2007] investigated the MAB mechanism under a relaxed restriction where advertisers have no incentive to deviate from a symmetric Nash equilibrium. Gatti et al. [2012] generalized previous results to a more detailed setting where externalities between multiple slots and different ads are considered.

6.3. Discussions

It is promising to combine mechanism design with online learning. Beyond the existing research works, we think the following directions are interesting and worth investigating.

First, most of the previous works view the MAB mechanism as a single-shot game on time period $[1, T]$, i.e., advertisers need to submit their bids at the beginning of the game and cannot change their bids during game. However, it is clear that many advertisers keep changing their bids in order to maximize their utilities. A dynamic version of the MAB mechanism will be more reasonable for sponsored search.

Second, as will be discussed in Section 10, advertisers' information is limited and they may behave irrationally. In this case, the truthfulness constraint in the MAB mechanism might not be necessary, and the search engine may also need to explore advertiser's (adversarial) bidding behaviors, in addition to exploring users' (stochastic) click behaviors. This actually poses a challenge to classic online learning: the reward is determined by both a stochastic factor and an adversary factor, which makes the online mechanism design highly non-trivial.

Third, most of the previous works define the action of the search engine as selecting an ad to display. However, this only corresponds to the ranking rule of the auction mechanism. It would be more appropriate to take the entire auction space (including both the ranking and pricing rules) as the action space for the search engine. This is not a simple action space enlargement, since one needs to leverage the structures in the new space. For examples, some ranking rules will produce the same allocation, and therefore their CTR information obtained through the exploration can be shared with each other.

7. AUCTIONS WITH DEPENDENT CTRS

In reality, the CTR of an ad depends on not only the ad itself but also the other ads shown in the same session. For example, by controlled experiments, Reiley et al. [2010] showed that additional rival mainline ads tend to increase the CTR of the ad allocated at the first mainline slot. Xiong et al. [2012] performed an analysis by looking at the CTRs of the same ad in the same position for the same query, but surrounded by different other ads. The finding is that in most cases the CTRs vary largely, indicating that the relationships between ads really impact their CTRs.

In the literature, people have introduced several click models to describe the dependency of CTRs, and revisited the fundamental issues regarding sponsored search auctions given these click models, such as the optimal auction mechanism design.

7.1. The Choice Model

Ghosh and Mahdian [2008] used a choice model to describe user click behaviors. In the model, a user gets exposed to a list of ads and clicks/converts on at most one of them. With this model, the optimization problem for the auctioneer is to choose a set S of at most k ads to maximize the expected social welfare, which can be written as a 0-1 integer program as below.

$$
\begin{aligned}
& \max_{x_i} \sum_{i \in [n]} v_i x_i Pr[\forall j = 0, 1, \cdots, n : q_i > q_j x_j] \\
s.t. \quad & \sum_{i \in [n]} x_i \leq k, \\
& x_i \in \{0, 1\}, \forall i \in [n] \\
& x_0 = 1,
\end{aligned}
$$

where x_i indicates whether bidder i gets a slot, v_i is the per-click value for bidder i, q_i is the ad-CTR of bidder i, q_0 is the threshold for the user to determine whether or not to take an action, and the probability is over a random draw of $(q_0, q_1, \cdots, q_n)$ from a joint distribution.

Ghosh and Mahdian [2008] showed that, the above optimization problem, even with no cardinality constraint (i.e., $k = n$), is NP-hard. Furthermore, the problem is hard to approximate in polynomial time within a factor $n^{1-\epsilon}$ for any $\epsilon > 0$, unless NP=ZPP. They then proposed two algorithms that can approximate the optimal solution within a factor of $\frac{e^2}{e-1}\lceil \ln R \rceil$, where R is an upper bound on the ratio between the highest value and the lowest value that a bidder could have. In some special cases, most notably when the distributions are single-peaked, they showed that the optimization problem can be solved exactly in polynomial time. These algorithms combined with a VCG-style payment scheme give rise to dominant-strategy incentive compatible mechanisms.

Note that the choice model assumes that the user behaviors are independent of the ranking order of the ads, which is a little inconsistent with our intuition. The cascade models described in the next subsection, instead, take the ranking order into consideration.

7.2. The Cascade Models

The cascade models have been studied in organic search [Craswell et al. 2008], and have been empirically verified by using impression and click-through data from commercial search engines [Gomes et al. 2009]. They have been introduced to sponsored search auctions in recent years [Aggarwal et al. 2008; Deng and Yu 2009; Giotis and Karlin 2008; Kempe and Mahdian 2008].

The basic cascade model for sponsored search auctions assumes the following user behaviors:

(1) Start with the ad in slot 1 (denoted as a_1).
(2) Having looked at the ad in slot i (denoted as a_i), click on it with probability q_{a_i}.
(3) Proceed to slot $i + 1$ with probability c_{a_i}; or terminate the scanning process with probability $1 - c_{a_i}$.
(4) Terminate the scanning process once there are no more ads left.

With this model, the CTR of the ad a_i can be written as

$$q_{a_i} \prod_{j=1}^{i-1} c_{a_j}. \tag{3}$$

On this basis, given an assignment a for n bidders and k slots, we can write the social welfare as follows,

$$SW(a) = \sum_{i \in [k]} v_{a_i} q_{a_i} \prod_{j=1}^{i-1} c_{a_j}.$$

Aggarwal et al. [2008] studied the above social welfare maximization problem and obtained several properties of the optimal allocation: (1) The most efficient allocation is no longer rank-by-revenue. (2) In the most efficient allocation, the ads are ranked in the decreasing order of $\frac{q_i b_i}{1-c_i}$, but the winners are not necessarily the top-ranked k ads. (3) For any bidder i that is assigned an ad slot in the optimal allocation, if some other bidder j is not in the allocation, and $q_j b_j \geq q_i b_i, \frac{q_j b_j}{1-c_j} \geq \frac{q_i b_i}{1-c_i}$, then replacing bidder i by j will get an allocation whose efficiency is no worse. (4) As a bidder increases his/her bid (fixing all the other bids), the probability for him/her to receive a click in the optimal allocation does not decrease, and his/her position in the optimal allocation does not go down. Based on these properties, they provide a dynamic program that can find an optimal allocation in a time complexity of $O(nlogn + k^2log^2n)$.

Deng and Yu [2009] studied a new ranking rule for GSP under the cascaded model, in which ads are ranked in the decreasing order of $\frac{q_j b_j}{1-c_j}$. They proved that there exists a pure Nash equilibrium in this auction and the efficiency of the Nash equilibrium is examined through both theoretical analysis and simulation. GSP with the same ranking rule is also studied in [Gomes et al. 2009], in which a Nash equilibrium is constructed that can maximize the search engine's revenue among all pure Nash equilibria.

On top of the basic cascaded model, a number of extensions have been investigated. For example, Giotis and Karlin [2008] added the slot-CTR (interpreted as the scanning continuation factor) θ_i to the cascaded model. Accordingly, the CTR of the ad in slot i can be written as

$$q_{a_i} \prod_{j=1}^{i-1} \theta_j c_{a_j}.$$

With this model, the GSP mechanism is studied when its ranking rule ranks ads in the decreasing order of $q_i b_i$, and its pricing rule charges the advertiser in slot i by $p_i = \frac{q_{a_{i+1}} b_{a_{i+1}}}{q_{a_i}}$. The following results are obtained from the study: (1) There exists a pure Nash equilibrium. (2) The price of anarchy of this auction against both VCG and the best GSP equilibrium is k in the restricted case (with conservative bidders), and is infinite in the unrestricted case.

Kempe and Mahdian [2008] further generalized the above model to allow for multiple separate blocks (e.g., the mainline ads and the sidebar ads), and proposed approximation algorithms to allocate the ads under this generalized model.

7.3. The Social Context Graph Model

Fotakis et al. [2011] used a social context graph to describe users' click behaviors. The graph depends on the advertisers' IDs, the relative order in the list of displayed ads, and the distance in the list. It assumes that users only focus on a window of consecutive slots, i.e., an ad can only influence its neighborhood ads in the ranked list.

They showed that the winner determination problem regarding the social context graph model is NP-hard. One can only get an exact algorithm of polynomial time complexity when the number of ad slot is small, however, a polynomial time approximation algorithm can always be designed in the general case. Furthermore, some negative results are obtained regarding the game-theoretic properties of the auction given the existence of the social context graph.

7.4. Discussions

Relaxing the assumption of independent CTRs is a significant step towards the real-world sponsored search auctions. The works reviewed in this section have made nice attempts along this direction. Going forward, there are several other promising directions that one may want to look into.

First, most works introduced in this section assume that the parameters in the click models are known to both the auctioneer and the bidders. In reality, however, these parameters need to be estimated, and the estimation could have errors (e.g., bias and variance). It is an interesting topic whether and how the estimation error of these parameters will impact the efficiency and revenue of the auction mechanism.

Second, the works reviewed in this section mostly focus on the allocation and efficiency analysis. It is unclear what kind of revenue one could obtain when the click models are used. It would be meaningful to study how to design revenue-maximizing mechanisms, or at least design some mechanisms with certain revenue guarantee.

Third, as mentioned above, due to the complexity caused by the dependent CTRs, the winner determination problem is usually NP-hard, and approximation algorithms are often needed. Those approximation allocations will lead to different equilibria from the optimal allocation. More investigations on this aspect will provide useful insights on the practical performance of the mechanisms.

Fourth, when the CTR of an ad is impacted by other ads, an advertiser may prefer to bid differently when surrounded by different ads. For example, he/she may be willing to bid and pay more if no direct competitor is shown in the same session. Therefore, it would make sense to provide a richer bidding language for advertisers to express their preferences (see Section 11). The interplay of new bidding languages and dependent CTRs would be a promising research direction.

Fifth, going beyond CTR externalities, value externalities have also attracted some research attention recently [Hoy et al. 2013; Ma et al. 2014] and are worthy of further investigation.

8. AUCTIONS WITH BUDGET CONSTRAINTS

In practice, budgets have been widely used by advertisers to cap their spending in a given period of time when running their ad campaigns. There are a number of reasons for an advertiser to use budgets. First, due to the dynamics of search users, advertisers will prefer to spend their money smoothly in order to reach more effective users. Second, budgets can help advertisers to control the risks of click fraud. Third, it is the main way for brand advertisers to control the expense of their ad campaigns.

In this section, we will introduce the existing works on sponsored search auctions with budget constraints. In particular, we will focus on the research from the perspective of the auctioneer, and will not cover the works on budget optimization for the advertisers [Feldman et al. 2007; Archak et al. 2010].

The general model for budgeted auctions can be described as follows. 1) There is a set of n advertisers and each advertiser i has a budget B_i. 2) There is a set of m keywords and each keyword is associated with a set of k ad slots. Each ad slot s is associated with a CTR θ_s. 3) Advertiser i has a private value $v_{i,j}$ on keyword j, and will reports his/her bid $b_{i,j}$. 4) The allocation rule is denoted by $x_{i,j,s}$: $x_{i,j,s} = 1$ if slot s of keyword j is allocated to advertiser i; and $x_{i,j,s} = 0$ otherwise. 5) The payment will be $p_{i,j,s}$. The total payment of advertiser i can be represented as $P_i = \sum_{j,s} p_{i,j,s}$. 6) The utility of advertiser i is a function of his/her total value for the allocated slots, the total payment, as well as the budget constraint.

We will introduce three lines of research in this section, each of which adopts some additional assumptions on the above general setting. The findings in these works are non-trivial, because the existence of the budget changes the quasi-linear nature of the utilities, and consequently significantly modifies the theory both technically and conceptually.

8.1. Budgeted Allocation

In the first line of research, one mainly cares about the optimal allocation but not the payment rule. Therefore it is usually referred to as *budgeted allocation*. The assumptions made by such research is as follows: the payment of a winner is simply assumed to be known (either set to be his/her bid price [Azar et al. 2008; Chakrabarty and Goel 2008] or to be an pre-estimated expected cost [Abrams et al. 2007]). Then the revenue extracted by the algorithm from advertiser i is the minimum of his/her budget B_i and the sum of the payments for keywords j allocated to him/her, i.e.,

$r_i = \min\{B_i, \sum_j p_{i,j}x_{i,j}\}$. The goal is to maximize the total revenue extracted from all the advertisers, i.e., $\max \sum_i r_i$, by treating the allocation $x_{i,j}$ as the only variables.

Azar et al. [2008] showed that the budgeted allocation problem is not only NP-hard but also APX-hard (i.e., there is no PTAS for this problem unless P=NP). Most of the existing works therefore try to find constant approximation algorithms. For example, Azar et al. [2008] assumed that there is only one ad slot per keyword ($k = 1$), and provided a $2/3$-approximation algorithm (and further improved this ratio to $\frac{1}{\sqrt{2}}$ for the uniform case of the problem). Chakrabarty and Goel [2008] improved the approximation ratio in [Azar et al. 2008] to $3/4$, and further proved that it is NP-hard to approximate the problem by any factor better than $15/16$. In [Abrams et al. 2007], multiple ad slots are allowed, and the budgeted allocation problem is formulated as an integer linear program. The problem is then approximated by means of fractional relaxation, and solved using the simplex method. Simulations show that this relaxation leads to quite good approximation in practice.

8.2. Single-Shot Auctions with Budget Constraints

In the second line of research, the focus is to analyze (different kinds of) equilibria when there are multiple slots but only one keyword (i.e., $m = 1$). In this case, the utility of advertiser i can be written as $u_i = \theta_s x_{i,s} v_i - p_{i,s}$, if $p_{i,s} < B_i; u_i = C$, otherwise, where C is some fixed value. Since this setting eliminates the repetitive nature of the auctions, we call it *single-slot auctions with budgets*. In this setting, the budget constraint is placed on a single keyword.

Ashlagi et al. [2010] proposed a modified generalized English auction (GEA) for advertisers with private values and private budget constraints. In particular, they defined C in the utility of advertiser i to be 0, and proved that the ex-post equilibrium outcome of the proposed auction is envy-free and Pareto-efficient. In addition, according to their proof, any auction mechanism that always obtains envy-freeness and Pareto-efficiency in ex-post equilibrium must choose the same slot assignment and the same payment as their proposed mechanism. Van der Laan and Yang [2011] also performed an equilibrium analysis by assuming private values and private budgets. The difference from [Ashlagi et al. 2010] lies in two aspects: (i) in the definition of the utility function for advertiser i, C is set to be $-\infty$ instead of 0; (ii) the CTRs of different slots are assumed to be identical, i.e., $\theta_s = 1$. In this setting, a new solution concept called *rationed equilibrium* is introduced, and an ascending auction mechanism is designed which always results in a rationed equilibrium in a finite number of steps. Kempe et al. [2009] adopted the same settings as in [Van der Laan and Yang 2011] and studied the price identification problem so as to ensure the envy-freeness of the allocation.

8.3. Multi-Unit Auction with Budget Constraints

The third line of research regards sponsored search auctions as multi-unit auctions, and performs mechanism design and equilibrium analysis when there are budget constraints.

The following settings are considered in these works: (1) identical keywords and single slot: $v_{i,j} = v_i, b_{i,j} = b_i$ and $m = 1$; (2) static bidding: the advertisers submit their bids at the very beginning and will not change their bids during the repeated auctions. In this setting, sponsored search auctions become auctions for multi-units of a single good, and the utility for advertiser i can be written as $u_i = X_i v_i - P_i$, if $P_i < B_i; u_i = C$, otherwise. Here X_i denotes the total number of keywords (more accurately, the ad slots associated with them) allocated to advertiser i, i.e., $X_i = \sum_j x_{i,j}$; P_i denote the corresponding total payment, i.e., $P_i = \sum_j p_{i,j}$; and C is a fixed value.

[Borgs et al. 2005] assumed that $C = -\infty$ in the utility function, and proved that it is impossible to design a non-trivial truthful auction that allocates all keywords, as long as the value is private. Then the hard condition of selling all keywords is relaxed, and an incentive-compatible mechanism is constructed which asymptotically achieves revenue maximization. Abrams et al. [2007] followed the same setting as in [Borgs et al. 2005], and studied the problem of revenue maximization. In particular, (i) they proved that the revenue of the optimal omniscient auction that sells keywords at many different prices is within a factor of 2 of the optimal omniscient auction that sells all the keywords at a single price; (ii) they designed a new auction mechanism that can obtain a constant competitive ratio when the advertiser dominance is large; (iii) they showed that no auction can guarantee $\frac{1}{2-\epsilon}$ of the revenue of the optimal omniscient multi-price auction; (iv) they designed an auction mechanism parameterized by the advertiser dominance that is constantly competitive in the worst case and is asymptotically optimal.

Dobzinski et al. [2008] also followed the settings in [Borgs et al. 2005] and studied the Pareto efficiency of the allocations. They proved a similar impossibility results as in [Borgs et al. 2005]. Then instead of studying approximation mechanisms, they investigated some special cases for which incentive-compatible Pareto optimal auctions can be found. In the first special case of their interest, the value of an advertiser is either much larger or much smaller than his/her budget. In the second special case, the budget information is publicly accessible. They showed that, in the latter case, the adaptive clinching auction is the only Pareto-optimal incentive compatible auction under certain conditions. Inspired by the negative results given by [Dobzinski et al. 2008], Hafalir et al. [2009] turned to design a mechanism named *Sort-Cut* that can achieve the so-called *semi-truthfulness* (i.e., it is a weakly dominant strategy for all advertisers to state their true budgets and not to understate their values). For their designed mechanism, they prove that some equilibrium can optimize the revenue over all Pareto-optimal mechanisms and this equilibrium is the unique one resulting from a natural rational bidding strategy (where every losing advertiser bids at least his/her true value). Bhattacharya et al. [2010] also followed [Dobzinski et al. 2008] and demonstrated the *Budget Monotonicity* property of the adaptive clinching auction.

Arnon and Mansour [2011] abstracted the existing mechanisms for multi-unit auctions with budgets, and studied their equilibrium and dynamics. In particular, they found that if the losing advertisers bid conservatively, then there always exists a pure Nash equilibrium. Furthermore, if the advertisers are myopic, the budgeted auction converges to a Nash equilibrium when there are two advertisers, or there are multiple advertisers with identical budgets.

Recently, some more complex settings of the budgeted auction are also studied. For example, Koh [2011] studied the budgeted auctions in the multi-slot setting. Specifically, they regarded slots as divisible goods, and identified a profile of budget thresholds for the existence of symmetric Nash equilibrium in this situation. Colini-Baldeschi et al. [2012] studied the setting of multiple keywords, and designed a mechanism which is incentive compatible in expectation, individually rational in expectation, and Pareto optimal in this setting. Furthermore, they gave an impossibility result for incentive compatible, individually rational, Pareto optimal, and deterministic mechanisms for advertisers with diminishing marginal values.

8.4. Discussions

In this section, we have introduced the impact of budgets on sponsored search auctions. There are still a lot of related issues that are not clear yet.

First, in almost all the previous works, the budget is regarded as a hard constraint. However, the fact is that budgets in sponsored search are usually softly implemented. Specifically, advertisers can change their budgets at any time. Even if the advertisers do not adjust their budgets, most commercial search engines allow certain overspending of the budgets (e.g., by 20%). Therefore, it would be better to reconsider the modeling of budget constraints, probably in a softer and more flexible manner.

Second, one of the major motivations for having budgets in ad campaigns is to smooth the ad spending, however, it might not be the only (or the best) way to do so. It is clear that advertisers can achieve smooth spending of their money through algorithmic adjustment of their bids. Actually many big advertisers do so in their daily ad campaigns. Another motivation for having a budget is to deal with the risk of click fraud. However, today commercial search engines provide very effective tools to detect click fraud and in addition allow advertiser to appeal (and get refunded) if they have evidence of click fraud. In this regard, it is not clear whether the budget is really a key feature of sponsored search auctions that we should study.

Third, all the settings used in the aforementioned papers are some kinds of simplification of the general setting. For example, many works assume a static bidding process, however, real advertisers may have strategic bidding behaviors along with time. As a result, it is unclear whether the conclusions drawn under the simplified assumptions have their direct values to the real sponsored search auctions. It would be important to discuss the extension/generalization of these results, e.g., by considering the dynamic behaviors of the advertisers.

9. BROAD MATCH AUCTIONS

In the previous sections, it has been assumed that, for a given query, only those ads that exactly bid on the query will be included in the auction. However, such an *exact match* setting might not be sufficient to satisfy the real requirements from commercial search engines, due to the following reasons. First, since the query space is extremely large (billions of queries are issued by web users every day), it is practically impossible for advertisers to exactly bid on every query related to their ads. Second, even if advertisers are capable enough to bid on the huge number of related queries, the search engine might not be able to afford it because of the scalability and latency constraints. Due to these reasons, commercial search engines usually adopt a *broad match* mechanism. A broad match mechanism requires advertisers to bid on at most κ keywords instead of an arbitrary number of queries, and matches the keywords to queries using a query-keyword bipartite graph, in which the number of keywords is significantly smaller than the number of queries. The broad match mechanism is friendly to advertisers since they only need to consider a relatively small number of keywords in order to reach the large volume of queries of their interests. The mechanism is also friendly to the search engine since it restricts the complexity of the bidding language as well as the auction system.

With the broad match mechanism, when a query is issued, all the ads bidding on the keywords that can be matched to the query on the bipartite graph will be put together for auction. And for every advertiser, the bids on the matched keywords will be transformed to the bid on the query using some pre-defined heuristics (e.g., the maximum bid on the matched keywords).

While the broad-match mechanism has its merits, it also brings in some issues. First, given the new structure of the auction (i.e., the query-keyword bipartite graph), one needs new techniques to perform optimization either from the search engine's perspective or from the advertisers' perspective. For example, Feldman et al. [2007] developed a $(1 - 1/e)$-approximation algorithm for the budget optimization problem. Even Dar et al. [2009] showed that the bid optimization problem in the broad-match setting is

NP-Hard and is inapproximable with any reasonable constant approximation ratio unless $P = NP$. Second, it becomes unclear whether the known theoretical properties of the GSP mechanism still hold in the broad-match setting. Several pieces of research [Mahdian and Wang 2009; Dhangwatnotai 2012; 2011; Chen et al. 2014] have been conducted to gain understanding on this topic. In the following subsections, we will make brief introductions to these theoretical studies.

9.1. GSP for Broad-Match Auctions

There have been several pieces of work in the literature that study the theoretical properties of GSP in the broad-match setting. For example, Mahdian and Wang [2009] showed that if the keyword set that an advertiser can bid on is pre-assigned, there may not exist a pure Nash equilibrium for the broad-match GSP mechanism. The non-existence of pure Nash equilibrium is also observed in [Dhangwatnotai 2012], under a more reasonable circumstance.

By assuming advertisers to play undominated strategies, Dhangwatnotai [2012] developed an almost-tight bound for the pure PoA of the broad-match GSP mechanism, based on the assumption that its pure Nash equilibrium exists. This PoA bound is dependent on two concepts, expressiveness and homogeneity. Expressiveness (denoted as α) describes the capacity of the κ keywords in covering the queries for which the advertiser has positive values through the bipartite graph. Homogeneity (denoted as c) describes the diversity of an advertiser's valuations on different queries matched to the same keyword on the bipartite graph. Take the keyword "spider" as an example. It can be matched to multiple queries, such as "spider movie", "get rid of spider", and "crystal spider", which have different semantic meanings. If each advertiser is only interested in one type of these semantic meanings, the homogeneity will be very large due to the high valuations on some queries and the low valuations on the other queries.

Dhangwatnotai [2011] proved that if the auction system is α-expressive and c-homogeneous, when a pure Nash equilibrium exists, the pure PoA of GSP is at most $\frac{c^2+c}{\alpha}$. When homogeneity c is large (which is usually the case as discussed above), this tight PoA bound becomes quite large, indicating that the social welfare of the broad-match GSP mechanism is not very good. In contrast, as mentioned in Section 4, in the exact-match setting, the pure PoA of GSP can be upper bounded by 1.282. This indicates that the introduction of broad match significantly affects the social welfare of GSP.

Some explanations on the above phenomenon are given in [Mahdian and Wang 2009], which basically attributes the problem to the heterogeneous nature of the broad match mechanism. That is, for each round of query auction, the mechanism puts together the ads bidding on (possibly very) different keywords. To tackle this problem, Mahdian and Wang [2009] proposed a clustering method to simplify the query-keyword bipartite graph such that each query is matched to at most one keyword. However, this approach seriously reduces the degree of broad match due to the removal of many query-keyword pairs in the bipartite graph, which is not desirable.

9.2. Designing New Mechanisms for Broad-Match Auctions

As mentioned in the previous subsection, the game-theoretic properties of GSP become much worse in the broad match setting. In this situation, it makes sense to design new auction mechanisms in order to obtain better theoretical properties.

Chen et al. [2014] proposed a new broad-match auction mechanism. The basic idea is that for a query that can be matched to multiple keywords on the bipartite graph, one assigns its impression in each round of auction to only one of these keywords in probability. Specifically, for each query, the new mechanism assigns a matching probability to every keyword that can be matched to it. When the query is issued by a user, the mechanism randomly samples a keyword according to the matching probability distribution, and then runs the GSP auction only upon those ads that bid on the sampled keyword. The new mechanism is called a *probabilistic broad match* mechanism. It is clear that this mechanism is not heterogeneous for any single round of auction due to the probabilistic sampling of keywords.

Chen et al. [2014] then performed a game-theoretic analysis on the probabilistic broad match mechanism, and obtained the following results. First, due to the removal of the heterogeneity in any single round of auction, there exists at least one pure Nash equilibrium for the probabilistic broad match mechanism. Second, an almost-tight pure PoA bound ($\frac{c}{\beta}$) is given for the social welfare in the Nash equilibrium, which is better than the pure PoA bound for the standard broad-match GSP in many practical situations. Third, a lower bound of the search engine revenue in envy-free equilibrium is given. This bound demonstrates that the worst-case revenue of the probabilistic broad match mechanism is better than the worst-case social welfare of the standard broad-match GSP under reasonable conditions.

9.3. Discussions

As mentioned in the beginning of this section, broad match has its great practical importance in sponsored search auctions. However, many existing studies have shown that the theoretical properties of GSP become unexpectedly bad when the broad match is enabled. Although there are some attempts on refining GSP in order to resolve these problems, the attempts are far from sufficient.

First, in practice, broad match and exact match are simultaneously supported by commercial search engines. That is, when a query comes in, both the ads that exactly bid on the query and the ads that can be matched to the query through the broad-match bipartite graph will be included in the auction. However, usually these two kinds of ads have different properties (in terms of their relevance, click probabilities, and bids) and will be treated in slightly different ways (and priorities). The existing studies on broad match have not taken this into consideration, and thus it is not very clear whether they can be naturally extended to this scenario. For example, it is unknown whether the theoretical results of the probabilistic broad match mechanism still hold when there are at least two keywords selected in each auction (one for exact match and the other for broad match).

Second, most of the theoretical results on broad match are obtained in the single-slot setting, because it is more difficult to analyze the multiple-slot setting. However, if we want to fully understand the impact of broad match on real sponsored search systems, it is necessary to consider the multi-slot setting.

Third, the existing theoretical studies on broad match are mostly concerned with GSP. It is unclear whether the broad match will have similar impact on other mechanisms, and whether one could design a new auction mechanism that is incompatible with GSP but has better theoretical and practical properties.

10. ADVERTISER BEHAVIOR MODELS

In auction theory, it is usually assumed that bidders' behaviors are fully rational. That is, bidders are assumed to have good knowledge about the auction mechanism, have well-defined utilities, be aware of who else are playing the game with them, have accurate guess of competitors' strategies, and be capable enough to take all these information into consideration so as to maximize their own utilities. However, in real sponsored search systems, these kinds of assumptions are not reasonable. In particular, 1) real advertisers have limited access to the information about their competitors since commercial search engines have the responsibility to protect the privacy of advertisers; 2) real advertisers have very little knowledge about the detailed implementation of the sponsored search system (e.g., ad selection, broad match, and click prediction), therefore could not have an accurate estimation of their utilities given a strategy; 3) real advertisers may be prevented by many factors, either subjectively or objectively, from taking their “best response” (e.g., constrained by budget, information collection cost, and opportunity cost). In this situation, the full rationality assumption might not provide the right insights, and one may need to revisit the auction mechanisms by appropriately modeling advertisers’ behaviors without full rationality assumption. In recent year, more and more people conduct their research along this direction.

10.1. Data Driven Advertiser Behavior Models

There have been a couple of works that build realistic advertiser behavior models based on auction logs. For example, Duong and Lahaie [2011] observed from data that advertisers see substantial variation in rank even if their bids are static. Thus, they bypass bids and directly model advertisers’ choices of rank, and proposed a *discrete choice model* to describe advertisers’ behaviors. Specifically, the utility of advertiser i obtained from position j can be written as:

$$u_{i,j} = (v_i - p_j)CTR_{i,j},$$

where v_i is the per-click value of advertiser i, p_j is his/her payment for the assigned position j, $CTR_{i,j}$ is the CTR for advertiser i if his/her ad is displayed at position j. The advertiser will select the next position corresponding to a higher utility with a larger probability, e.g., $Pr(i,j) = \frac{e^{\lambda u_{i,j}}}{\sum_{k=1}^{K} e^{\lambda u_{i,k}}}$. Note that all the variables in this model are observable for advertisers, and parameter v_i can be learned from data by regression. Sodomka et al. [2013] generalized the discrete choice model across terms and campaigns and proposed a predictive model for advertisers’ values on different keywords. However, in these works, it is not specified how advertisers can achieve their next targeted positions by appropriately setting their bids.

Pin and Key [2011] proposed an *analytic model* to describe advertisers’ behaviors. This model is motivated by the observations on real data that advertisers usually enter many distinct auctions with different opponents and with varying parameters. In this situation, it is more appropriate to assume that advertisers set their bids to optimize the expected utilities (suppose they know the bid distributions of their opponents). If we assume bidders to behave according to this model, we can derive a necessary condition for the relationship between bids and true values, which can be used to learn the values or to predict future bids. A problem with this model lies in that it might not be reasonable to assume advertisers to know others’ bid distributions, especially when all the bidders are maximizing their expected utilities in complicated situations.

Xu et al. [2013] and He et al. [2013] adopted parametric models to learn advertisers’ behaviors. Xu et al. [2013] modeled the bounded rationality of advertisers from three dimensions: willingness, capability, and constraints. Willingness describes how likely an advertiser tends to change his/her bid; capability measures the ability of an advertiser to collect information, estimate his/her competitors’ strategies, and find his/her best response; constraints reflect whether an advertiser is free to take his/her best response, without being constrained by his/her remaining budgets and other factors. The parameters in this rationality model can be learned from data and the model can then be used to predict advertisers’ bids in the future auctions. He et al. [2013] assumed that each advertiser changes his/her bid only based on some local information (e.g., his/her own campaign performance in the past period of time, his/her remaining budget, etc), and proposed a Markov behavior model to describe how an advertiser changes his/her bid given the local information in a probabilistic manner. The parameters in the Markov model can be learned from data based on simple statistical and machine learning methods. They then proposed a bi-level learning framework to obtain the revenue-maximizing auction mechanism when advertisers are not fully rational but behave according to the Markov model.

10.2. Discussions

Considering the large number of advertisers and their diverse behaviors in real sponsored search, it is a big challenge to accurately model and predict the behaviors of individual advertisers.

First, different types of advertisers may have very different behaviors. For example, some head advertisers (such as Ebay and Amazon) usually invest hundreds of millions of dollars in search marketing, and they have their dedicated teams or vendor companies to optimize the ad campaigns. In this case, it is reasonable to assume that they have very good knowledge about the search engines and their competitors, and are highly capable of optimizing their strategies. In contrast, most tail advertisers have little budget on search advertising, and are usually inexperienced and incapable of maximizing their utilities. On the other hand, there are a number of ad agencies in the market, who can help medium or small advertisers to run their ad campaigns. These agencies can accumulate certain knowledge about how search engines run the auctions, and can coordinate the strategies of different advertisers to achieve some *global* optimization. With the existence of the ad agencies, the behaviors of some advertisers are no longer independent, but instead in a corporative or coopetitive manner. As a result, in order to better model advertisers' behaviors, one may need to categorize advertisers into different types (which by itself is a challenging topic) and handle them with different techniques.

Second, advertisers' behaviors sometimes can be influenced by the search engine. As we know, search engines usually provide a set of tools to advertisers, including keyword suggestion, traffic prediction, and bid optimization. Advertisers who adopt these tools and those who work by their own usually have different behaviors. It would be useful to take this aspect into consideration when constructing advertiser behavior models. In addition, suppose all the advertisers will use the tools provided by the search engine and follow the suggestions, the search engine actually has an opportunity to create some correlated equilibrium, rather than the natural Nash equilibrium, so as to achieve an even better social welfare or revenue.

Third, sponsored search auctions involve different micro markets. These micro markets can be characterized by the keywords, the categories of products, and/or the regions and countries. Advertisers' behaviors should be quite different for different micro markets. For example, for the keywords related to popular and highly profitable products, advertisers may be highly aggressive and proactive; and for some other keywords, advertisers may be risk averse and conservative. For another example, due to the huge differences in cultures across different regions and countries, the behavior model learned from a US advertiser may fail with a large probability when applied to a Chinese advertiser. Therefore, it is important to accurately partition the keywords and advertisers into micro markets, which is, however, not an easy task.

Fourth, in real sponsored search systems, advertisers manage their ad campaigns through a hierarchical bidding structure. That is, advertisers need to create their accounts, campaigns, ad groups, and finally individual ads and keywords. Budget constraints are usually imposed at the campaign level, and bids are usually set at the ad group level. However, as far as we know, no previous work has explicitly considered this hierarchical structure when building advertiser behavior models. We believe that it would be beneficial to consider this structure due to the following reasons: 1)

Advertisers' decisions on different levels may demonstrate some common properties, since they are made by the same person for the same campaign goal; 2) Decision for sibling nodes in the hierarchy, e.g., for different ad groups under the same campaign, may be correlated with each other. 3) For the same ad group, bids for different match types may be correlatedly different. On one hand, since these bids are for the same keyword, they will demonstrate certain correlation. However, since exact-match and broad-match usually have different marketing effects, advertisers will have different values associated with them. Therefore the corresponding bids will also be different even if the keyword is the same.

Fifth, the bidding behaviors of real advertisers may also be affected by signals outsides the auction system. For example, the day of the week, the season of the year, the occurrence of some specific event, and the global trend of the economics environment may all have their impacts on the behavior of an advertiser. If one wants to build an accurate advertiser behavior model, these external signals should not be neglected. However, since the external signals come from an open world, it is very challenging to achieve a comprehensive consideration. Not to mention that deep domain knowledge is sorely needed to get a clear picture of what kind of signal is relevant and what is not.

11. AUCTIONS WITH RICH BIDDING LANGUAGES

Most previous works assume that an advertiser only submits a single bid representing his/her willingness to pay. In recent years, many researchers have realized the limitation of such a single-dimensional bidding language, and have started to study more complicated settings where the bidding language can be more expressive.

11.1. Conditional Bids

The works introduced in this subsection do not only allow advertisers to submit bids to express their willingness to pay, but also allow them to specify the conditions for the bids to take effect.

Aggarwal et al. [2007] allowed an advertiser to specify the positions (more accurately the top κ_i positions) that he/she is interested in (which we call prefix range for ease of reference). Only if his/her ad is eventually allocated in the prefix range, his/her bid will take effect and the search engine can charge him/her. In this setting, Aggarwal et al. [2007] proposed a top-down auction mechanism, which works in the following manner. For each position (ordered from top to bottom), one iteratively runs a simple second-price auction (with one winner) among those advertisers whose prefix ranges include the position being considered. Here, a "simple second-price auction" means that (1) the highest bidder in the auction gets the position, and pays a per-click price equal to the second-highest bid; (2) the winner is then removed from the pool of advertisers for subsequent auctions. It is easy to see that similar to GSP, this mechanism also ranks the ads in the decreasing order of b_i, and if a user clicks on an ad, the advertiser will pay the minimum amount that he/she would have needed to bid in order to keep the current rank position. Furthermore, it can be shown that the auction has an envy-free (or symmetric Nash) equilibrium with the same outcome as VCG, and this equilibrium is the best such equilibrium for the advertisers in terms of their utilities.

Muthukrishnan [2009] allowed an advertiser to specify the maximum number l_i of other ads that can be shown together with his/her own ad. In other words, if the search engine displays more than l_i ads to a user, this advertiser will prefer to be excluded from the auction. A mechanism for this setting should determine the number of ads to show, the ranking of the ads and the payment of each winner of the auction. In [Muthukrishnan 2009], a simple mechanism that meets these requirements is proposed, which can find the optimal number of slots and the optimal assignment in $O(nlog^2n)$ time.

Constantin et al. [2011] allowed advertisers to specify a set of constraints on his/her rank position relative to the positions of certain other advertisers (for example, an advertiser may only accept the allocation of a slot higher than his/her direct competitors). If the constraints are not satisfied, the advertisers will refuse to pay. In this setting, Constantin et al. [2011] proposed a natural extension of GSP, called expressive GSP (eGSP), which induces truthful revelation of constraints for a rich subclass of unit-bidder types. They then proved the existence of envy-free equilibrium under eGSP with a further restriction to a subclass of exclusion constraints, for which the standard GSP has no pure Nash equilibrium. In addition, they proved that the winner determination problem in this situation is NP-hard, and proposed a greedy allocation algorithm with bounded approximation ratio for social efficiency.

11.2. Multiple Bids

The works introduced in this subsection allow advertisers to submit multiple bids, to reflect their valuations on different outcomes of the auction.

In [Ghosh and Sayedi 2010], an advertiser is allowed to submit two bids: one stands for the value when only his/her own ad is displayed, and the other corresponds to the value when several other ads are also displayed in the same session. The auctioneer has to decide whether to display one or more ads, how to allocate the slots, and how to compute the corresponding payments. Ghosh and Sayedi [2010] studied two GSP-inspired mechanisms in this setting: (1) GSP_{2D}, which is designed with the property that the allocation and payment are identical to GSP when multiple ads are shown; (2) NP_{2D}, which is designed to be a next-price auction. They then compared the efficiency and revenue in equilibrium for the two mechanisms: the results are rather subtle, depending on whether the efficient output is a single ad or includes multiple ads, and conditioned on the bidding strategies of the losers.

Hoy et al. [2013] proposed a so-call *coopetitive ad auction* model. In this model, an advertiser can derive values from the clicks on other ads, and therefore needs to submit multiple bids to reflect these values. Formally, for each ad j, there is a publicly-known set of advertisers S_j who all derive value from a click on the ad, and advertiser i derives the same value v_i from a click on any ad j where $i \in S_j$. Hoy et al. [2013] showed that in the single-slot setting, standard solutions, including the status quo ignorance of mutual benefit and a benefit-aware VCG mechanism, perform poorly. However, an appropriate first-price auction has nice equilibria: all the equilibria that satisfy a natural cooperative envy-freeness condition select the welfare-maximizing ad and satisfy an intuitive lower-bound on revenue.

Goel and Munagala [2009] extended GSP to a truthful hybrid auction in which an advertiser can submit both a per-impression bid and a per-click bid. Different from most works on sponsored search auctions, which treat sponsored search auctions as single-shot games, they viewed sponsored search auctions as repeated games. The setting in their work is as below.

— There are n advertisers compete for a single slot associated with a specific keyword.
— An advertiser j arrives at time $t = 0$; he/she submits a per-impression bid $m_{j,t}$ and a per-click bid $c_{j,t}$ for the auction at time t.
— There is a publicly known ad-CTR $q_{j,t}$ for advertiser j at time t.

Then the hybrid auction mechanism calculates the effective bid of each advertiser as $b_{j,t} = \max(m_{j,t}, c_{j,t}q_{j,t})$ and allocates the ad slot to the advertiser with the largest effective bid. Let j^*, t denote the advertiser who wins the auction and $b_{-j^*,t}$ denote the second largest effective bid in auction t. If $m_{j^*,t} > c_{j^*,t}q_{j^*,t}$, the winner pays a per-impression price $b_{-j^*,t}$; otherwise, he/she pays a per-click price $\frac{b_{-j^*,t}}{q_{j^*,t}}$.

Edelman and Lee [2008] proposed another hybrid auction mechanism which allows an advertiser to submit a pay-per-click bid and a pay-per-action bid at the same time. Dominant-strategy equilibria, Bayesian Nash equilibria, and their implications to the revenue of the search engine are then discussed with respect to this hybrid auction mechanism.

11.3. Discussions

In this section, we have introduced some works that go beyond single dimensional bids. Actually, the requirements from real applications on the bidding language can be even more complicated and expressive than the ones introduced above. For example, as the sponsored search industry continues to evolve, ads with rich formats gradually emerge: ads can now display additional data, include site links, click-to-call phone numbers, images and videos; take up more or less space; and allow social and other forms of interactions. There are many challenges to address when designing the bidding language and auction mechanism in this new scenario. We make some discussions along this line as follows.

First, considering the setting that an ad can take up multiple slots, it is a challenging problem how to characterize a good mechanism. For example, it needs to be discussed whether the bidding language should support advertisers to bid for the number of slots, and whether it is possible to achieve efficient allocation of the slots to the advertisers. One intuition is that the relaxation of the sizes of the ads may make the allocation problem NP-hard, and one may have to design approximate algorithms for the allocation and pricing.

Second, rather than the one-dimensional layout where ads are placed from top to bottom, ads can also be placed in a two-dimensional manner. This has already happen in today' touch devices. In this new setting, it becomes unclear what the corresponding bidding language should become and how the optimal allocation is defined. It is also difficult to specify the slot-CTR due to the lack of mature user studies on the click behaviors/patterns given the two-dimensional layout.

Third, if heterogeneous ads (e.g., five text ads with site links, three image ads, and two video ads) participate in the same auction, it is unclear how the bidding language should be designed and how the auction should be executed. The user actions (and therefore the payment conditions) will also be different. For example, triggered by a mouse move, an image ad may be enlarged, and a video ad may be played. As a result, the auction mechanism should consider all these different user actions in a harmony manner. In one word, there are quite a lot of open questions regarding the rich formats of sponsored search auctions, waiting for researchers to study and resolve.

12. CONCLUSIONS

Sponsored search auctions have attracted a lot of research attention in recent years because of its great business success. We believe a comprehensive survey of this area will be helpful for both researchers and practitioners. This is exactly the motivation for us to write this paper.

In this paper, we have categorized existing research into two categories according to the assumptions they use.

— The works with basic settings assume that the advertisers are fully rational, have no budget constraint, their ads have known and independent click-through rates, and queries exactly match the bid keywords of the advertisers. In this paper, we have made detailed introductions to the game-theoretic analysis made by these works on equilibrium concepts, social efficiency, and revenue.

— The works on advanced topics relax one or several assumptions made by the basic settings. This category of research can model real-world sponsored search auctions in a more accurate manner, however, due to the relaxed assumptions, its theoretical analysis becomes more challenging. In the paper, we have introduced existing attempts along this line, and have also listed quite a few future research directions, hoping that they could be inspiring for readers to look into related fields and make their own contributions to the community.

Overall speaking, we believe an ultimate solution to sponsored search auctions in real settings should involve multiple disciplines such as game theory, machine learning, and data mining. We highly encourage people with different backgrounds to collaborate with each other, and advance the state of the art by working together.

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